To convert a set of three Euler angles, $\phi_1$, $\phi_2$, $\phi_3$ (1, 2, 3 are the first, second, third Euler rotations, not the axes of rotation) to the equivalent quaternion:

Note: You must know the Euler rotation axis sequence, i.e 123, 321, 213, 121, etc.

1) form three quaternions from the three Euler angles:
   a. for a “1” rotation axis, the quaternion is $\sin(\phi/2) 0.0 0.0 \cos(\phi/2)$
   b. for a “2” rotation axis, the quaternion is $0.0 \sin(\phi/2) 0.0 \cos(\phi/2)$
   c. for a “3” rotation axis, the quaternion is $0.0 0.0 \sin(\phi/2) \cos(\phi/2)$

2) multiply the three quaternions in the correct order.

for example,

given:

rotation order 312
$\phi_1 = 30 \text{ deg}$
$\phi_2 = 60 \text{ deg}$
$\phi_3 = 45 \text{ deg}$

$Q_1 = 0.0 0.0 \sin(30/2) \cos(30/2) = 0.0 0.0 0.258819045 0.965925826$
$Q_2 = \sin(60/2) 0.0 0.0 \cos(60/2) = .5 0.0 0.0 0.866025404$
$Q_3 = 0.0 \sin(45/2) 0.0 \cos(45/2) = 0.382683432 0.0 0.0 0.923879533$

$Q_f = Q_1 Q_2 Q_3$
$= 0.360423406 0.43967974 0.391903837 0.723317411$